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SURFACE WAVES IN LATERALLY INHOMOGENEOUS REGIONS

by

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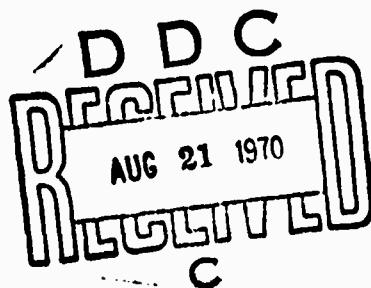
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Seismic data will allow us to infer the physical properties of the interior of the earth only if three conditions are met:

- a) the nature of the release of seismic energy is understood,
- b) the effect of propagation through an arbitrary transmitting medium can be calculated, for a class of media spanning the possible range of true conditions, and
- c) a unique relationship can be found between the physical properties of the medium, and the propagation of stress waves through that medium.

None of these conditions is satisfied for real seismic sources and the real earth. Therefore, seismologists interpret seismic data in terms of simple source models and earth models for which the above conditions hold. In this work we have extended the class of models for which those conditions hold so that the models may describe reality more closely.

a) Seismic Sources

It has previously been assumed that an earthquake focus could be represented by an edge dislocation along a plane fault surface. This source produces displacements identical to a double couple set of body forces, and is thus referred to as the double couple source model. The double couple source may be described in terms of pure shear strain, and involves no net force nor net moment. Such a source is consistent with the notion that the energy released in an earthquake is identically that produced by relative displacement across a fault. Because displacement is known to occur across fault surfaces, the double couple model is appealing on physical grounds for surface earthquakes.

At some depth in the earth, almost certainly within the crust, the shear stress needed to overcome friction across an arbitrary surface exceeds the shear strength of the rock. Below this depth, the concept of a "fault" becomes meaningless, and the double couple model loses some of its physical appeal for those earthquakes occurring in the mantle. A competing model has been developed (Knopoff and Randall, 1970) to describe the effects of a rapid phase change. Rapid changes in density and/or bulk modulus produce an isotropic radiation pattern, with radial first P-motions and no first S-motions. A rapid change in shear modulus would result in a double couple radiation pattern if

the initial stress were homogeneous plane shear; however, given an axial strain field before the event, the radiation pattern would be that of a linear vector dipole. The change in axial strain resulting from the phase change requires a volume change, thus, the total radiation field would be the sum of the isotropic radial field and the linear vector dipole field. This model will be referred to as the "Compensated Linear Vector Dipole."

In Randall and Knopoff (1970) the effects of propagation through a layered, attenuating earth model with a crust are considered. A statistical test is given for discriminating between the double couple, the compensated linear dipole, and the simple volume change models, based on amplitudes of P wave first arrivals. This method is applied to five deep focus earthquakes. For most of the shocks, the double couple seems to be the dominant source mechanism, but there is evidence that the compensated linear vector dipole plays a significant role (i.e., 65% double couple, 35% CLVD).

b) Wave Propagation

We have made progress in extending the complexity and generality of earth models for which surface wave velocity dispersion curves can be computed. Previously these earth models have been limited to perfectly elastic layered half

spaces. We have significantly increased program efficiency, allowing more detailed models to be processed in a given amount of computer time, we have generalized the theory so that attenuation in the various layers may be considered explicitly, and we have developed a transformation which allows the layered half space programs for Love wave dispersion to be applied to layered spheres.

Each point on a dispersion curve represents a solution of an eigenvalue problem involving the structural parameters and the two variables,  $k$ , the wave number, and  $\omega$ , the frequency. The several methods for computing dispersion curves differ only in their representation of the "dispersion function," a matrix product which enters into the eigenvalue equation in  $k$  and  $\omega$ . Although the various representations are equal when fully expanded, the speed of computation and degree of physical insight provided depend strongly on the compressed form of the representation. We have shown that the various representations fall into two categories (Schwab, 1970)

"The Thomson-Haskell technique synthesizes, or builds up the surface-wave dispersion functions by constructing layer matrices which relate the components of motion at one interface in a layered structure to those at the next. The product of these layer matrices then relates the components

of motion at the deepest interface to those at the free surface, and this layer-matrix product is used to construct the dispersion function. Knopoff's technique begins with the immediate construction of the dispersion function in its full determinantal form, and then analyzes, or decomposes the determinant into a product of interface matrices, which are derived from submatrices of the determinant. Each of these interface submatrices relates the components of motion in the layer on one side of the interface to those in the layer on the other side."

"Randall (1967) performed the initial programming of Knopoff's technique for Rayleigh waves and demonstrated the validity of the method. During the recent work on the optimization of the computational aspects of the Thomson-Haskell formulation (Schwab and Knopoff, 1970), it became evident that much the same optimizing processes could be applied to Knopoff's technique, and that they would lead to considerable simplification in the computation of the Rayleigh-wave dispersion function, as well as improving the speed of computation. Two modifications of the original form of Knopoff's technique bring about these improvements."

"In its initial form the method employed interface-matrix elements which were complex. A simple change in notation allows all elements to be expressed as either pure real or pure imaginary quantities. Using the technique given

by Schwab and Knopoff, this allows all computations to be carried out with pure real numbers. This eliminates the extra time required to compute matrix products when complex elements are involved. It also removes the problems inherent in bracketing and refining roots of a dispersion function having both real and imaginary parts, since this function now becomes a pure real quantity."

"The second change merely requires that the elements of the interface matrices, which are defined as fourth-order determinants for Rayleigh-wave computations and second-order determinants for Love wave computations, be evaluated analytically before numerical computations begin. In the final representation of the Rayleigh-wave dispersion function derived from Knopoff's method, numerical evaluation of the matrix elements directly from the determinantal definitions leads to significant loss in speed of computation, since four of the elements in each interface matrix are seen to vanish when evaluated analytically. The expressions for the other elements are quite simple, thus considerable computation time is saved by elimination of these determinantal definitions from the actual program."

The modifications for Rayleigh-wave dispersion computations were shown to increase the speed of the Thomson-Haskell technique by a factor of almost 60, based on a test in which the original and modified programs were used

to compute dispersion curves for the same model (17 layers plus half space) on the same machine. This improvement should help to reduce computation costs for those involved in wave propagation studies under the VELA project. The modified Thomson-Haskell method and the Knopoff method compared as follows (Schwab, 1970):

"Of the various versions of the Thomson-Haskell formulation for Rayleigh wave dispersion computations, the reduced-delta-matrix extension is the most powerful, i.e., the fastest which contains the feature controlling the loss-of-precision problem occasionally encountered by the original formulation. The results of the delta-matrix extensions are actually contained in Knopoff's work, which appeared earlier than these extensions, and these results are obtainable directly from his formulation without recourse to delta-matrix theory. The flexibility of Knopoff's method is used to devise a new representation of the Rayleigh wave dispersion function which is more powerful than the most powerful of the Thomson-Haskell versions, i.e., it contains the loss-of-precision control feature and is about 38% faster than the reduced-delta-matrix extension; and in fact, is about 12% faster than the fastest of the Thomson-Haskell versions."

"Knopoff's method is particularly suitable for Rayleigh wave dispersion computations involving the use of both

solid and liquid layers. The technique for the construction of solid-solid interface matrices can be generalized, in an obvious manner, to the construction of solid-liquid, liquid-liquid, and liquid-solid interface matrices. The only difference between Knopoff's method and the Thomson-Haskell technique for the computation of Love wave dispersion is in the matrix representations of the dispersion function. The dispersion functions themselves, except for a possible sign reversal, the characteristic times, and the actual program speeds are all identical."

The introduction of complex elastic moduli in the layers allows the effects of attenuation to be considered directly. Preliminary results show that for a frequency-independent Q model typical of those in the literature, a reduction in phase velocity of less than  $10^{-3}$  km/sec occurs. This validates the usual assumption that the direct effects of attenuation on phase velocity are not detectable seismically. The method also allows direct computation of the attenuation factors for Love and Rayleigh waves as a function of period. Preliminary results indicate that the upper mantle acts as a "notch filter;" waves of certain periods are almost undamped, while neighboring periods are strongly attenuated. This effect may explain many of the puzzling aspects of higher mode surface wave observations, such as the propagation of Lg waves across continents but not oceans.

Biswas and Knopoff (1970) have developed a transformation which allows Love wave dispersion curves for a layered half-space to be converted for use on a layered sphere. Although the equations of motion for the toroidal oscillations of a sphere may be solved directly, the half-space to sphere conversion offers certain advantages: first, dispersion curves for a layered half space can be computed much more rapidly, by the methods outlined above, than those for the layered sphere, and second, the conversion method allows calculation of the phase velocity at frequencies other than those corresponding to the integral toroidal modes. Normal mode frequencies have been calculated from the converted phase velocities, and are indistinguishable from those obtained by the exact spherical computations. The ability to compute dispersion curves as a continuous function of frequency is especially important when determining group velocity curves. The group velocity  $U$  is obtained by differentiating a function of the phase velocity,  $c$ , with respect to frequency:

$$1/U = dw(\omega/c)/d\omega .$$

Where  $c$  is known only at widely separated, discrete points, differentiation is not justified, and the group velocity will not be known even at those discrete points. Interpolation by standard procedures can help only slightly. The earth flattening transformation, however, provides a

continuous phase velocity curve from which the group velocity curve may be obtained by differentiation.

c) Inversion

Theoretical progress in the inversion of non-linear systems of equations, and the developments of rapid dispersion curve computations mentioned above, now permit us to find the range of reasonable earth models which could produce seismically observed surface wave velocities. Without introducing arbitrary restrictions on the models considered, unique earth structures cannot be found. However, the range of possible structures consistent with the experimental data can be explored. We are currently applying the so-called "hedgehog" technique. A criterion of fit is selected, consistent with the accuracy of the data, and a model is found by steepest descent, Monte Carlo search, or guesswork which satisfies the criterion of fit. A set of parameters is then established to describe possible models, and the parameters are then incremented to produce new models. Dispersion curves for those models are then compared with the seismic data, and the process is repeated for successful models. Thus, the "volume" in parameter space of the satisfactory models is systematically eaten away, as implied by the term "hedgehog." The importance of rapid dispersion calculations is evident.

We are presently investigating extensions of the inversion scheme which will determine the number of equivalent degrees of freedom in a set of geophysical data and select the optimum set of data points from continuous data.

d) Applications

The surface wave dispersion techniques described above have been applied to Rayleigh waves across the East Pacific Rise. In each case the "one station method" was used, requiring that the earthquakes studied have vertical strike slip sources. The intent of these studies was to limit the propagation path to a well-defined region, so that the dispersion curves would not represent averages over quite different structures. For this reason, rather small shocks were studied. The earthquakes were in the Rivera fracture zone, a transform fault between two sections of the East Pacific Rise, so the shocks were almost certainly vertical strike slips. The stations were located in Culiacan and La Paz. Inversion was performed both by standard least squares procedure (Knopoff, Schwab and Schlue, 1970) and by the "hedgehog" method (Knopoff, Schlue and Schwab, 1970).

The path from the Rivera Fracture Zone to Culiacan was characterized by a transitional oceanic structure with a well

developed low velocity zone for shear waves. The lid velocity was around 4.4 to 4.45 km/sec, while the channel velocity was 4.1 km/sec. The top of the low velocity zone occurred at about 50 km. below sea level. The data did not extend to long enough periods to determine the depth of the bottom of the low velocity zone. The Rivera Fracture Zone to La Paz path showed rather unusual character. The phase velocity decreased slightly with increasing period, in the range 25-65 seconds. The least squares inversion produced a structure with a very thick lid, 90 km, and an extremely low channel velocity of 3.5 km/sec. Another model in satisfactory agreement with the data has a decrease in velocity with depth at 50 km, and another decrease at 90-100 km. The velocity in the deep channel is 4.3 km/sec. In any case, there is evidence of a strong anomaly along the path from Rivera Fracture Zone to La Paz.

#### Summary

Significant advances have been made in the degree of sophistication of the earth models and fault models to which standard geophysical analysis may be applied. An alternate to the double couple fault model has been developed, and is called the compensated linear vector dipole. This source model is appropriate for a rapid phase change in a region

of uniaxial stress. A statistical test has been devised to distinguish between the two alternate source models. Several deep focus earthquakes have been examined with this test. The double couple source appears predominant, although the compensated linear vector dipole is also significant.

Tremendous improvements have been made in the speed with which dispersion curves can be computed for layered half space earth models. Attenuation in the layers can now be included explicitly. A transformation from the flat layered half space to the layered sphere makes use of the fast dispersion calculations for the half space, and allows calculation of phase velocities at frequencies other than those of the normal modes of the earth.

The "hedgehog" procedure has been developed and programmed for the inversion of surface wave dispersion curves. This systematically explores the set of earth structures compatible with the data.

The above techniques have been applied to data for Rayleigh waves across the East Pacific Rise. The path between the Rivera Fracture Zone and Culiacan appears similar to normal oceanic transitional mantle. The path from the Rivera Fracture zone to La Paz seems anomalous, with an ultra-low velocity zone beginning at about 100 km. depth.

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13. ABSTRACT <p>A new model for a seismic source mechanism has been developed. The new model describes the action of a rapid phase change as a seismic source and is an alternate hypothesis to the double couple model. Great improvements have been made in the efficiency of programs for calculating surface wave dispersion curves for a layered half space. Attenuation in the layers may be taken into account directly. A transformation has been developed which maps Love wave dispersion curves for a layered half space into layered sphere dispersion curves, thus allowing use of the more efficient half space dispersion programs. Hedgehog inversion systematically identifies a range of acceptable solutions to a system of nonlinear equations. The above developments have been employed to analyze Rayleigh wave dispersion across the East Pacific Rise.</p>		
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